

COMPLEX DYNAMICS IN ODD-DEGREE POLYNOMIAL MAPS

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Abstract. Complex dynamics is detected in a class of maps of odd-degree of linearity. It illustrates saddle connections, fractalization of basins with self-similarity and chaotic attractors. This paper describes these dynamic behaviors, bifurcations, and chaos. Fractals basins are displayed by numerical simulations.

Keywords. Invariant set; Endomorphism; Attractor; Bifurcations; Manifold.

AMS (MOS) subject classification: 37G10, 37J20, 37J15, 37D10

1 Introduction

We deal with two-dimensional noninvertible maps which can contribute greatly to the understanding of complex nonlinear dynamics. Many dynamic behaviors have been studied extensively, particularly in the applied cubic dynamics literature, and constitute a central issue in bifurcation theory (Agarwal in [1]; May in [12]). The main purpose of this paper is to show some mechanisms associated with invariant curves. We shall see that a mechanism, that may be considered as typical in some classes of maps of odd degree of linearity, is associated with a saddle-connection, also called heteroclinic loop defined as a closed invariant curve formed by the merging of a branch of stable set of a point of a saddle cycle with the unstable branch of another point of another saddle, thus forming a closed connection among the points of saddles. These structurally unstable situations cause bifurcations between different dynamic behaviors. As these kinds of bifurcations cannot be predicted by local investigations, they can be classified as global bifurcations. A cubic regular-uncoupled model is considered, which we will not discuss the "uncoupled" character in this work. It is of interest and offers a richness of bifurcations and an interesting set of dynamical phenomena due to the presence of multistable states, deformation of basin boundaries and transient chaos.

An attractor is a set towards which a dynamical system evolves over time. It is very common for dynamical systems to have more than one at-