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ON SOME THIRD-ORDER DIFFERENCE EQUATIONS

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Abstract. In this paper, we obtain symmetries and solutions of difference equations of the form

$$u_{n+3} = \frac{u_n}{A_n + B_n u_n u_{n+1} u_{n+2}}$$

where $(A_n)_{n\in\mathbb{N}_0}$ and $(B_n)_{n\in\mathbb{N}_0}$ are sequences of real numbers. This work generalises a result of Abo-Zeid. Furthermore, our method, which involves Lie point symmetry analysis, is different from the technique used by Abo-Zeid.

Keywords. Difference equation, symmetry, reduction, group invariant, solution.

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1 Introduction

Lie Symmetry Analysis can be used as a method for solving differential equations. The method has been applied in recent times to difference equations (See [10, 7, 9, 5, 6]). In this method, one finds the group of transformations that leave the difference equation under study unchanged. The calculations can become more tedious as the order of the equation increases.
In [8], khalaf Allah studied the asymptotic behaviour of solutions to the rational difference equations

$$x_{n+1} = \frac{x_{n-2}}{\pm 1 + x_n x_{n-1} x_{n-2}}. \tag{1}$$

Abo-Zeid discussed global asymptotic stability of all solutions of the difference equation

$$x_{n+1} = \frac{A x_{n-2}}{B + C x_n x_{n-1} x_{n-2}}, \tag{2}$$

where A, B, C are positive real numbers [1]. Our goal is to give exact solutions of the following difference equations via the invariant of their group of