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A NON-CONSISTENT BOUNDARY VALUE PROBLEM OF A GENERALIZED LINEAR DISCRETE TIME SYSTEM

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Abstract. In this article we study a class of generalised linear systems of difference equations with given boundary conditions and assume that the boundary value problem is non-consistent, i.e. it has infinite many or no solutions. We take into consideration the case that the coefficients are square constant matrices with the leading coefficient singular and provide optimal solutions. Numerical examples are given to justify our theory.

Keywords. generalised, system, difference equations, linear, discrete time system.

1 Introduction

Many authors have studied generalised discrete & continuous time systems, see [1–18], advanced differential equations, see [19–23], and their applications, see [24–36]. Many of these results have already been extended to systems of differential & difference equations with fractional operators, see [37–48]. We consider the generalised discrete time system of the form

$$FY_{k+1} = GY_k, \quad k = 1, 2, ..., N-1 \tag{1}$$

with known boundary conditions

$$A_1 Y_0 = B_1, \quad A_2 Y_N = B_2. \tag{2}$$

Where $F, G \in \mathbb{R}^{r\times m}$, $Y_k \in \mathbb{R}^m$, and $A_1 \in \mathbb{C}^{r_1\times m}$, $A_2 \in \mathbb{C}^{r_2\times m}$, $B_1 \in \mathbb{C}^{r_1\times 1}$ and $B_2 \in \mathbb{C}^{r_2\times 1}$. The matrices F, G can be non-square ($r \neq m$) or square ($r = m$) with F singular (det$F$=0).

Generalised linear systems of difference equations with given boundary conditions don't always guarantee to have unique solution. In the case where there exist solutions and they are infinite, we require optimal solutions for the system. The aim of this paper is to generalise existing results regarding the literature. An explicit and easily testable formula is derived of an optimal solution for the system.

All authors have contributed equally to the article.