Dynamics of Continuous, Discrete and Impulsive Systems
Series **A:** *Mathematical Analysis* **28 (2021)** 377-392
 http://www.watam.org

ASYMPTOTIC BEHAVIOR OF PERTURBED DIFFERENTIAL EQUATIONS USING NON-SMOOTH LYAPUNOV FUNCTION

M.A. Hammami[1] and N. Hnia[2]

[1] Department of Mathematics
University of Sfax, Faculty of Sciences of Sfax,Route Soukra, BP 1171, Tunisia

[2] Department of Mathematics
University of Sfax, Faculty of Sciences of Sfax,Route Soukra, BP 1171, Tunisia

Abstract. Most Lyapunov stability theorems presented in the literature require that the Lyapunov function candidate for a nonlinear dynamical system be a continuously differential function with a negative definite derivative, this is due to the fact that the majority of the dynamical systems considered are systems possessing continuous motions, and hence, Lyapunov theorems provide stability conditions that do not require knowledge of the system trajectories. However, for some dynamical systems such as biological systems and impulsive systems one can take a Lyapunov function candidate which not necessarily differential which arise naturally. In this paper, we give some new sufficient conditions for the asymptotic or exponential stability of a class of nonlinear time-varying differential equations. We use the Lyapunov method with functions that are not necessarily differential where we extend some previous results.

Keywords. Lyapunov functions, Functions bounds, comparison lemma, Asymptotic stability.

AMS (MOS) subject classification: 45J05, 45D05, 26D10, 45M10.

1 Introduction

Time- varying differential equations appear as a natural description of observed evolution phenomena of certain real world problems where the study of asymptotic stability is more important than stability. The qualitative behavior of the solutions of perturbed nonlinear systems of differential equations is often studied by obtaining a Lyapunov function for the unperturbed system and using it as a Lyapunov function for the perturbed system. It is well known that the second method of Lyapunov ([1], [2]) provides sufficient conditions to ensure various types of stability for a dynamical system described by ordinary differential equations with perturbations (see [3] - [9]). The perturbation term could result from errors in modeling a nonlinear system, aging of parameters or uncertainties and disturbances. In general, we know some