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DYNAMICS OF A HIGHER ORDER NONLINEAR RATIONAL DIFFERENCE EQUATION

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Abstract. In this article, we investigate the periodicity, the blondeness and the global stability of the positive solutions of the following nonlinear rational difference equation in a higher order:

$$x_{n+1} = \left(\prod_{i=1}^{N} x_{n-k_i}\right) / \left(\sum_{i=1}^{N} \alpha_i x_{n-k_i}\right), \quad n = 0, 1, 2, ...$$

where the parameters $\alpha_i \in (0, \infty), (i = 1, 2, ..., N)$, while $k_i (i = 1, 2, ..., N)$ are positive integers, such that $k_1 < k_2 < ... < k_N$, with $k_1 = 0$. The initial conditions $x_{-k_N}, x_{-k_N+1}, ..., x_{-k_2}, x_{-k_2+1}, ..., x_{-1}, x_0 \in (0, \infty)$.

Keywords. Difference equations; Higher order nonlinear rational difference equations; periodicity; blondeness; Global stability.

AMS (MOS) subject classification: 39A10, 39A11, 39A99, 34C99

1 Introduction

The topic of difference equation is interesting and attractive to many mathematicians working on this field. It is a fertile research area. Many real-life phenomena are modelled using the difference equations. Examples from economy, biology, etc. may be obtained in [11, 14, 16]. The study of some properties of these equations via the global attractively, the blondeness, and the periodicity are of great interest. For example in the articles [8, 9, 14, 15] closely related to global convergence results are obtained which can be applied to nonlinear difference equations in proving that every solution of these equations converges to a period two solution.
The objective of this article is to investigate some qualitative behavior of the solutions of the following higher order nonlinear rational difference equation:

$$x_{n+1} = \left(\prod_{i=1}^{N} x_{n-k_i}\right) / \left(\sum_{i=1}^{N} \alpha_i x_{n-k_i}\right), \quad n = 0, 1, 2, ... \tag{1}$$