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OSCILLATION THEOREMS FOR EMDEN-FOWLER TYPE DELAY DYNAMIC EQUATIONS ON TIME SCALES

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Abstract. We study the necessary and sufficient conditions for the oscillation of a class of second order half-linear dynamic equations of the form:

$$[a(t)(x^{\Delta}(t))^{\alpha}]^{\Delta} + \psi(t)x^{\beta}(\delta(t)) = 0$$

on time scales. The significance of our result is that α, β are quotients of odd positive integers without any restrictive conditions. Some examples are given to illustrate our main results.

Keywords. Oscillation, nonoscillation, dynamic equation, time scales, fixed point theorem.

AMS (MOS) subject classification: 34C10, 34K11, 34N05, 39A10.

1 Introduction

The main goal of the work is to study the necessary and sufficient conditions for oscillation of solutions of second order nonlinear dynamic equation on time scales of the form:

$$[a(t)(x^{\Delta}(t))^{\alpha}]^{\Delta} + F(t, x(\delta(t))) = 0,\ t \in \mathbb{T}_0 = [t_0, \infty) \cap \mathbb{T}, \tag{1}$$

where $\mathbb{T}$ is an arbitrary time scale with $\sup \mathbb{T} = \infty$. Throughout, we always assume that

($\mathcal{H}_1$) α is a ratio of odd positive integers, $\delta \in C_{rd}(\mathbb{T}_0, \mathbb{T})$ such that $\delta(t) \leq t$ and $\delta(t) \to \infty$ as $t \to \infty$;

($\mathcal{H}_2$) $a(t) \in C_{rd}(\mathbb{T}_0, \mathbb{R}_+)$ and $\int_{t_0}^{\infty} (a(s))^{-1/\alpha} \Delta s = \infty$.
Let $\mathcal{R}(t) = \int_{t_0}^{t} (a(s))^{-1/\alpha} \Delta s$. Then we have $\lim_{t \to \infty} \mathcal{R}(t) = \infty$;

($\mathcal{H}_3$) $F(t, x) \in C(\mathbb{T}_0 \times \mathbb{R}, \mathbb{R})$, $xF(t, x) > 0$ for $x \neq 0$, there exists $\psi(t) \in C_{rd}(\mathbb{T}_0, \mathbb{R}_+)$, $\psi(t) \not\equiv 0$ such that $|F(t, x)| \geq \psi(t)|x|^{\beta}$ and β is a quotients of odd positive integers.