

FRACTIONAL ORDER LINEAR TIME-DELAY DIFFERENTIAL INEQUALITIES AND APPLICATIONS

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Abstract. In this paper, we investigate fractional order linear differential inequalities with time-delay. Firstly, we extend the classical Halanay inequality with time-delay to fractional order. Next, an upper bound solution of fractional order Halanay inequalities for one dimensional and higher dimensional are obtained by using the contradiction method and M-matrix, respectively. Finally, we also study the Mittag-Leffler stability of fractional order Halanay inequalities and show sufficient pure algebraic stability criteria.

Keywords. Fractional order; inequality; Time-delay; M-matrix; Mittag-Leffler stability.

1 Introduction

The concept of fractional calculus can be traced back to more than 300 years [1], but it did not attract a great attention and research until the last few decades. With the advancement of technology, a large number of practical problems related to fractional derivatives have appeared in various disciplines and engineering practices. It is found that the fractional derivative is more accurate and effective than the classic integer derivative because of its memory and hereditary properties. At present, fractional derivative modeling is widely used in many fields such as physics, mechanics, chemistry, engineering, materials, finance, etc., which makes the theory of fractional order differential equations has become a new hot research field [2]-[12].

The study of stability theory is an important subject in the theory of fractional order differential equations. In 1996, Matignon D [13] discussed the stability of a class of fractional order linear systems. In 2009 and 2010,