

DYNAMICALLY CONSISTENT NONSTANDARD FINITE DIFFERENCE SCHEMES FOR A VECTOR-HOST EPIDEMIC MODEL WITH NONLINEAR INCIDENCES

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Abstract. In this work, we formulate and analyze nonstandard finite difference (NSFD) schemes for a recognized vector-host epidemic model with nonlinear incidences. The theoretical results and numerical simulations show that the constructed NSFD schemes preserve essential qualitative properties of the model. These properties include positivity and monotone convergence of solutions, equilibria and their stability. Especially, the stability of the proposed NSFD schemes is investigated based on an extension of the Lyapunov stability theorem. As an important consequence of this, we obtain dynamically consistent NSFD schemes for the epidemic model. In addition, we prove that standard finite difference schemes including the Euler scheme, the second order Runge-Kutta scheme and the classical fourth order Runge-Kutta scheme provide numerical solutions which are completely different from the solutions of the continuous model. Lastly, a set of numerical examples are performed to illustrate and support the obtained theoretical results.

Keywords. Nonstandard finite difference schemes; Lyapunov stability theorem; Positivity; Vector-host epidemic model; Dynamics consistency.

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1 Introduction

Mathematical models of infectious diseases have played an important role both in theory and practice, they are often used to predict and manage potential outbreaks (see, for instance, [2, 3, 6, 7, 8, 9, 29, 36, 37]). These models have attracted the attention of many mathematicians, biologists, ecologists and epidemiologists in various aspects, especially in the construction and analysis of mathematical models for particular diseases.

In this paper, we consider a recognized vector-host epidemic model with nonlinear incidence proposed in [9] and described by a system of nonlinear