

## QUALITATIVE BEHAVIOUR OF UNBOUNDED SOLUTIONS OF NEUTRAL DELAY DIFFERENTIAL EQUATIONS

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**Abstract.** In this paper, an attempt has been made to study the qualitative properties of unbounded solutions of fourth order homogeneous and nonhomogeneous equations under suitable assumptions. Examples are included to illustrate the validation of the results.

**Keywords.** Oscillation, unbounded solution, higher-order, neutral delay differential equations, asymptotic behavior.

**AMS (MOS) Subject Classifications:** 34 C 10, 34 C 15, 34 K 11.

### 1 Introduction.

In [10], Parhi and Tripathy have studied the oscillatory and asymptotic behaviour of solutions of

$$(r(t)(y(t) + p(t)y(t - \tau)))'''' + q(t)G(y(t - \alpha)) = 0 \quad (1)$$

and

$$(r(t)(y(t) + p(t)y(t - \tau)))'''' + q(t)G(y(t - \alpha)) = f(t) \quad (2)$$

respectively under the assumption

$$(A_0) \quad \int_0^\infty \frac{t}{r(t)} dt < \infty,$$

where  $p \in C([0, \infty), \mathbb{R})$ ,  $r, q \in C([0, \infty), (0, \infty))$ ,  $f \in C([0, \infty), \mathbb{R})$ ,  $G \in C(\mathbb{R}, \mathbb{R})$  and  $G$  is nondecreasing with  $uG(u) > 0$  for  $u \neq 0$ ,  $\tau > 0$  and  $\alpha > 0$ . Their work showed that if  $q(t) < 0$ , then it would be possible to obtain analogous results for oscillatory and asymptotic behaviour of solutions of (1) and (2). The problem remains open as to what happens if  $q(t)$  changes sign. In particular, if  $q(t) = q^+(t) - q^-(t)$ , where  $q^+(t) = \max\{0, q(t)\}$  and  $q^-(t) = \max\{0, -q(t)\}$ , then (1) and (2) can be viewed as

$$(r(t)(y(t) + p(t)y(t - \tau)))'''' + q^+(t)G(y(t - \alpha)) - q^-(t)G(y(t - \alpha)) = 0 \quad (3)$$