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NATURAL BOUNDARY CONDITIONS FOR A CLASS OF GENERALIZED FRACTIONAL VARIATIONAL PROBLEM

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Abstract. In this paper, we analyze a problem of the generalized fractional calculus of variations with free boundary values. Specifically, we focus on the generalized fractional variational problems with (i) finite subsidiary conditions, (ii) simple variable end point, (iii) end points lying on the given curves. For all of the mentioned problems, we derive the necessary optimality conditions. The formulation of these conditions involves Riemann-Liouville and Caputo's fractional order derivatives. With free terminal points in a simple variable end point problem, we obtain the natural boundary conditions that provide the additional information on the boundaries of the concerned problem. Furthermore, we derive the transversality conditions for the fractional variational problem with end point lying on the given curves. Both the natural boundary conditions and transversality conditions provide the missing boundary conditions for the variational problem with variable terminal points.

Keywords. Fractional derivatives, Fractional integrals, Euler-Lagrange equations, Natural boundary conditions, Transversality conditions

AMS (MOS) subject classification: 26A33, 49K20, 49M05

1 Introduction

The problem of the fractional calculus of variations (FCOV) is concerned with optimizing quantities involving fractional order operators. It is considered to be an extension of classical calculus of variations (COV) by allowing the order of derivative to be any real number. Non-integer order derivatives have found numerous applications in various fields such as optimal control theory, signal processing, viscoelasticity, anomalous transport and diffusion models, etc. Due to the non-local behavior, fractional order operators are suitable for modeling of systems with the previous data attached. The existence of fractional order derivatives was discussed by Leibnitz in 1695. For fundamental theory and evolution of fractional order operators (derivative or integral), we refer the reader to [15, 21, 24].