ON SOLVABILITY OF A SYSTEM OF THREE DIFFERENCE EQUATIONS

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Abstract. In this paper, we show that the following system of nonlinear difference equations

$$x_{n+1} = \frac{x_n z_n + a}{x_n + z_n}, \ y_{n+1} = \frac{y_n x_n + a}{y_n + x_n}, \ z_{n+1} = \frac{z_n y_n + a}{z_n + y_n}$$

for $n \in \mathbb{N}_0 = \mathbb{N} \cup \{0\}$ where the parameter a and the initial values x_0, y_0, z_0 are real numbers, can be solved in explicit form. Also, we investigate the asymptotic behavior of the well-defined solutions by using these formulas.

Keywords. Asymptotic behavior; explicit solution; forbidden set; nonlinear difference equation; system.

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1 Introduction

There has been an increasing interest in studying of nonlinear difference equations and their systems for the last two decades. For some examples of nonlinear difference equations, see [1, 10, 15] and for some examples of systems of difference equations, see [3, 4, 5, 7, 8, 9, 11, 13, 14, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27] and the references therein. The reason for this interest is undoubtedly the emergence of these equations in many disciplines such as biology, pyhsics, economics, etc. For some example, see the books [2, 6].

Li and Zhu [12] studied the globally asymptotic stability of the nonlinear difference equation

$$x_{n+1} = \frac{x_n x_{n-1} + a}{x_n + x_{n-1}} \tag{1}$$

for $n \in \mathbb{N}_0$, where $a \in [0, \infty)$ and the initial values are positive real numbers.

Yilmazyildirim and Tollu [28] obtained solution forms of the system of nonlinear difference equations

$$x_{n+1} = \frac{x_n + y_n}{1 + x_n y_n}, \ y_{n+1} = \frac{y_n + z_n}{1 + y_n z_n}, \ z_{n+1} = \frac{z_n + x_n}{1 + z_n x_n} \tag{2}$$