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ON A TIME-DISCRETE APPROACH TO THE NONLINEAR SCHRÖDINGER EQUATION

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Abstract. This paper is devoted to the large time behavior and especially to the regularity of the global attractor for the time-discrete Crank-Nicolson scheme to discretize 1D nonlinear Schrödinger equation with nonlocal integral term on $\mathbb{R}^1$. We first prove that such a time-discrete equation provides a discrete infinite dimensional dynamical system in $H^1(\mathbb{R}^1)$ that possesses a global attractor $\mathcal{A}_\tau$ in $H^1(\mathbb{R}^1)$. We also show that the global attractor $\mathcal{A}_\tau$ is regular, i.e., $\mathcal{A}_\tau$ is actually included, bounded and compact in $H^{\frac{3}{2}-\varepsilon}(\mathbb{R}^1)$. Furthermore we obtain the finite fractal dimension of $\mathcal{A}_\tau$. Moreover, we show that individual solution trajectories and the global attractor for the time-discrete nonlocal Schrödinger equation converge to the solution trajectories and the global attractor of the time-discrete classical Schrödinger equation, as the coefficient of the nonlocal integral term goes to zero.

Keywords. Nonlinear Schrödinger equation; Nonlocal integral term; Crank-Nicolson scheme; Global attractor; Regularity; Fractal dimension; Attractor convergence.

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1 Introduction

In this paper, we are interested in the large time behavior, and especially the regularity of the global attractor for the following time discrete 1D nonlocal nonlinear Schrödinger (NLS) equation on $\mathbb{R}^1$

$$\begin{cases} iu_t + u_{xx} + |u|^2 u + \gamma u \displaystyle\int_{-\infty}^{x} |u(y,t)|^2 dy + i\alpha u = f(x), & t \in \mathbb{R}_+, x \in \mathbb{R}^1, \quad (1) \\ u(x,0) = u_0(x), & x \in \mathbb{R}^1, \quad (2) \end{cases}$$

where the damping parameter $\alpha > 0$ and the dispersion parameter $\gamma \in \mathbb{R}^1$, the unknown function $u = u(x;t)$ is complex and the complex function $f(x)$; $x \in \mathbb{R}^1$ is known. Eq.(1) models the influence of ion inertia upon the nonlinear Langmuir waves on the basis of the Zakharov equations that perturb the classical NLS equation

$$iu_t + u_{xx} + |u|^2 u + i\alpha u = f. \tag{3}$$