Dynamics of Continuous, Discrete and Impulsive Systems
Series **B:** *Applications & Algorithms* **28 (2021)** 183-196

THE GRONWALL'S INEQUALITY ON THE (q, h)-TIME SCALE

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Abstract. In this article, we present an analogue of Gronwall's inequality on the (q, h)-time scale. Some particular cases are derived where discrete Mittag-Leffler functions are used. Using the inequality, we study the dependence of the solution on the order and initial condition of the fractional difference equation.

Keywords. Time scales; discrete time scales; (q, h)-time scale; Gronwall's inequality; Mittag-Leffler function.

AMS (MOS) subject classification: 39A70; 39A12; 39B05.

1 Introduction

The theory of discrete fractional calculus has received much interest in recent years. The foundation of this theory was formulated in pioneering works by Agarwal [3] and Diaz and Osler [15] (see also [18]), where basic approaches, definition, and properties of the theory of fractional sums and differences were studied. The extension of basic notions of fractional calculus to other settings was performed in [13], where fractional sums and differences studied in the framework of (q, h)-calculus, which can be reduced to ordinary difference calculus and q-difference calculus (see, e.g., [1, 2, 21]). Further work on (q, h)-calculus can be found in [14] where the discrete analogue of Mittag-Leffler functions in linear fractional difference equations are introduced. Recently, a series of papers continuing this research has appeared (see, e.g. [5, 6, 7, 22, 23, 24]).

The integral inequalities which are considered as an effective tool for qualitative analysis of the solutions to differential and integral equations have been also under consideration. In particular, we are interested in Gronwall's inequality which has been the main target for many researchers (see, e.g. [8, 10, 17, 19, 20]). On the other hand, the Gronwall's inequality on time scales are explored in [16, 25, 26, 27]. To the best of the authors' observation, however, the (q, h)-analogue of Gronwall's inequality has not been investigated yet.