

# HYBRID IMPLICIT FRACTIONAL INTEGRO-DIFFERENTIAL EQUATIONS WITH HADAMARD INTEGRAL BOUNDARY CONDITIONS

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**Abstract.** In this paper, we discuss the existence of solutions for a hybrid fractional integro-differential equations with Hadamard integral boundary conditions. we use the fixed point theorem due to Dhage to prove the main result. An example illustrating the existence result is also presented, in the second approach using while trying to weaken the conditions imposed Mönch fixed point theorem combined with technique of measure of non-compactness we study the existence and structure of solution sets.

**Keywords.** Banach algebra, Fractional differential equation, Hadamard fractional derivative, Hybrid differential equation, Dhage fixed point theorem.

**AMS (MOS) subject classification :** 26A33, 34A08, 34B15, 45J05, 34K05.

## 1 Introduction

Fractional differential equations has a large application in a variety of fields such as electrical networks, signal and image processing, viscoelasticity, aerodynamics, economics and do so on, hence has increased more attention from both theoretical and the applied points of view in recent years (for further details see [11, 15]). We notice here that most of the work on the topic in the literature is based on Riemann-Liouville and Caputo type fractional differential equations, for this we can refer the readers to [2, 3, 8, 9]. Another kind of fractional derivative appears side by side to RiemannLiouville and Caputo derivatives in the literature is Hadamard fractional derivative introduced in 1892 [14], which differs from the preceding ones in the sense that the kernel of the integral (in the definition of Hadamard derivative) contains logarithmic function of arbitrary exponent. Details and properties of Hadamard fractional derivative and integral can be found in [5, 6, 7, 16, 17, 18]. In recent years, quadratic perturbations of nonlinear differential equations have attracted much attention. We call such differential equations hybrid differential equations. There have been many works on the theory of hybrid differential equations, we refer the readers to the articles [1, 4, 19, 21]. Integro-differential and integrals equations of fractional order have also proved to be valuable tools in the modelling of many phenomena in various fields of science and engineering, the kind of this area of research have been studied by many authors [12, 13, 20]. Especially in [1] the authors discussed the existence of