

## EXISTENCE OF SOLUTIONS FOR NON-LINEAR HADAMARD FRACTIONAL DIFFERENTIAL EQUATION WITH MIXED FRACTIONAL BOUNDARY CONDITIONS

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**Abstract.** This paper is concerned with the existence of solutions for Hadamard fractional differential equation along with boundary conditions, including Fractional integral and Fractional derivative. In the present work, we convert the system into an equivalent integral equation and associate an operator with that integral equation. The results are based on the applications of Banach fixed point theorem, Krasnoselskii's fixed point theorem, Leray-Schauder's nonlinear alternative, Leray-Schauder degree in Banach space. Some examples are given to validate the results.

**Keywords.** Hadamard Fractional Derivative, Hadamard Fractional Integral, Boundary Value Problem, Fixed Point Theorems.

**AMS (MOS) subject classification:** Primary 26A33; Secondary 34B15.

### 1 Introduction.

Fractional differential equations have gained considerable importance due to their application in various sciences, such as physics, mechanics, chemistry, engineering. The interest of the study of fractional-order differential equations lies in the fact that fractional-order models are more accurate than integer-order models; that is, there are more degrees of freedom in the fractional-order models. Furthermore, fractional derivatives provide an excellent instrument for the description of memory and hereditary properties of various materials and processes due to the existence of a memory term in a model.

In recent years, most of the work is based on the fractional derivative of Riemann-Liouville and Caputo-type, for a detailed description, see [8, 14], and references therein. There is also another kind of fractional derivative that appears in the literature along with Riemann-Liouville and Caputo derivatives is the Hadamard fractional derivative due to Hadamard introduced in

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