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A FINITE HORIZON OPTIMAL STOCHASTIC IMPULSE CONTROL PROBLEM WITH A DECISION LAG

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Abstract. This paper studies an optimal stochastic impulse control problem in a finite time horizon with a decision lag, by which we mean that after an impulse is made, a fixed number units of time has to be elapsed before the next impulse is allowed to be made. The continuity of the value function is proved. A suitable version of dynamic programming principle is established, which takes into account the dependence of state process on the elapsed time. The corresponding Hamilton-Jacobi-Bellman (HJB) equation is derived, which exhibits some special feature of the problem. The value function of this optimal impulse control problem is characterized as the unique viscosity solution to the corresponding HJB equation. An optimal impulse control is constructed provided the value function is given. Moreover, a limiting case with the waiting time approaching 0 is discussed.

Keywords. Impulse control, decision lag, dynamic programming, viscosity solution, diffusion processes.

AMS (MOS) subject classification: 93E20, 49L20, 49L25, 49N25.

1 Introduction

Let $(\Omega, \mathcal{F}, \mathbb{F}, \mathbb{P})$ be a complete filtered probability space on which a d-dimensional standard Brownian motion $W(\cdot)$ is defined, with $\mathbb{F}$ being its natural filtration augmented by all the $\mathbb{P}$-null sets. Consider the following stochastic differential equation (SDE, for short):

$$X(s) = x + \int_t^s b(\tau, X(\tau))d\tau + \int_t^s \sigma(\tau, X(\tau))dW(\tau) + \xi(s), \quad s \in [t, T], \tag{1.1}$$

where $b : [0, T] \times \mathbb{R}^n \to \mathbb{R}^n$ and $\sigma : [0, T] \times \mathbb{R}^n \to \mathbb{R}^{n\times d}$ are some suitable deterministic maps, $X(\cdot)$ is the *state process* with $t \in [0, T)$ being the *initial time* and $x \in \mathbb{R}^n$ being the *initial state*, and $\xi(\cdot)$ is called an *impulse control* of the following form:

$$\xi(s) = \sum_{i\geqslant 1} \xi_i \chi_{[\tau_i, T]}(s), \quad s \in [t, T]. \tag{1.2}$$