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ON SOME PROPERTIES IN THE PARAMETER SPACE OF A PLANAR DISCRETE DYNAMICAL SYSTEM

Mohamed R. Ferchichi[1] and Abla Yousfi[2]

Department of Mathematics, Laboratory of Mathematics, Dynamics and Modelization,
University Badji Mokhtar, Annaba - Algeria
[1]Email: mrferchichi@yahoo.fr, [2]Email: doc.abla.yousfi@gmail.com

Abstract. The planar dynamical system considered in this paper is defined by the quadratic polynomial map $(x, y) \longmapsto T(x, y) = (ay + bxy, cx + y^2)$ where a, b and c are real parameters. We are particularly interested in the behavior of this system in its (a, b, c)-parameter space. In the (a, b_0, c)-parameter planes, where b_0 is any value of the parameter b, we prove the existence of symmetries with respect to their origin and their diagonals. These symmetries correspond, in the phase plane, to cycles of the same period, type and stability. We also prove the existence of a correspondance between cycles of the same period, type, stability and pairs of parameter values belonging to a same curve $a.c = constant$. On the other hand, we study the geometry of the Arnold tongues issuing from the Neimark-Sacker bifurcation manifold associated with the stability loss of the focus $(0, 0)$ of T.

Keywords. discrete dynamical systems, k-cycles, attractors, Neimark-Sacker bifurcation, Arnold tongues.

AMS (MOS) subject classification: 34D45 , 37G35, 37D45.

1 Introduction

The discrete dynamical system, considered in this paper, is generated by a quadratic polynomial map $T : \mathbb{R}^2 \to \mathbb{R}^2$, $(x, y) \longrightarrow (x', y')$, given by:

$$T: \begin{cases} x' = ay + bxy \\ y' = cx + y^2 \end{cases}, \qquad (1)$$

where a, b and c are real parameters.

In a previous paper [5] we studied, in the phase plane, the dynamics of this system for different values of its parameters. In particular, we studied two chaotic attractors, one of the Cantor-type and arising from a cascade of period-doubling bifurcations, the other arising from the complex evolution of an invariant closed curves cycle of period 2. This previous work motivated