QUADRATIC FORMS RELATED TO THE VORONOI’S DOMAIN FACES OF THE SECOND PERFECT FORM IN SEVEN VARIABLES

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Abstract. A new algorithm for calculating the Voronoi’s domain faces of the second perfect form is proposed in the paper and on its basis all the related quadratic forms of 27-dimensional faces of the Voronoi’s domain are calculated.

Keywords: quadratic form, arithmetic minimum, integer point, permutation.

1 Introduction

Modeling the classical Voronoi’s problem of finding perfect forms is closely related to the well-known Hermite problem of arithmetic minima of positive quadratic forms. These problems are interesting and non-trivial problems of geometric number theory, studied by many mathematicians [1, 2]. They are considered in the works by S.L. Sobolev in connection with the construction of lattice optimal cubature formulas. All this testifies to the urgency of the Voronoi’s problem to find perfect forms.

2 Modeling the classical Voronoi’s problem to find perfect forms

In [3], it was proved that quadratic forms corresponding to \( \frac{(n+2)(n-1)}{2} \)-dimensional faces \( \psi_k = \psi_k (\varphi^n_1) \) of domain \( V^{n(n+1)} \) of perfect form (p.f.),

\[
\varphi^n_1 = \varphi^n_1 (x_1, \ldots, x_n) = x_1^2 + \ldots + x_n^2 + x_1 x_3 + \ldots + x_{n-1} x_n
\]

have the form

\[
-x_1 x_3,
\]

\[
x_1 x_2 - \delta_{34} x_3 x_4 - \ldots - \delta_{n-1} x_{n-1} x_n
\]