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LYAPUNOV TYPE INEQUALITIES FOR FRACTIONAL DIFFERENTIAL IMPULSIVE SYSTEMS

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Abstract. The goal of this paper is to study fractional order differential impulsive systems and fractional impulsive Sturm-Liouville type problems derived from these systems. The main purpose is to obtain Lyapunov type inequalities for mentioned problems. As application of these inequalities, we investigate some qualitative properties such as stability, disconjugacy, nonexistence and oscillatory behaviour of fractional differential systems and fractional S-L type problems with impulsive conditions. We use the concept of fractional conformable operators to study fractional order Hamiltonian systems.

Keywords. Fractional derivatives and integrals, Differential impulsive system, Lyapunov type inequalities, Stability, Disconjugacy

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1 Introduction

The historical perspective of Lyapunov inequalities turns back to the last decade of nineteenth century. In 1892, A. M. Lyapunov introduced an effective tool for studying the qualitative behavior of second order differential equations with ω-periodic coefficients of the form

$$y'' + q(t)y = 0, \quad -\infty < t < \infty. \tag{1.1}$$

Lyapunov stated the following theorem.

Theorem 1.1. *[16][Chapter III, Theorem II] If the function q takes only positive or zero values(without being identically zero), and if further it satisfies the condition*

$$\omega \int_0^\omega q(t) \leq 4,$$

then roots of the characteristic equation corresponding to (1.1) will always be complex and their modulus are equal to 1.