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CONVEX OPTIMIZATION ON RIEMANNIAN MANIFOLDS

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Abstract. In this paper we extend some of the properties of convex optimization problems from Euclidean space to that of Riemannian manifold. We consider an optimization problem on a Riemannian manifold where the objective function is convex, the feasible set is totally convex but the functions attached to inequality constraints are not necessarily convex. We show that if the program is superconsistent and a non-degeneracy condition is satisfied then the Karush-Kuhn-Tucker (KKT) type optimality conditions are both necessary and sufficient. We construct a counter example on a Riemannian manifold in support of our results. We study some duality results for the Lagrange dual problems on these nonlinear spaces.

Keywords. Riemannian manifold, Convex optimization, KKT conditions, Slater constraint qualification.

AMS (MOS) subject classification: 58A05, 58B20, 26B25, 90C25, 90C46.

1 Introduction

Convexity plays an important role in optimization theory as any local minimum point of any convex objective function is also a global minimum point. Also there are many functions which are not convex but we can make them convex with the help of proper Riemannian metrices. Here we give one such example. We consider the Rosenbrock banana function [10], $f : \mathbb{R}^2 \to \mathbb{R}$, defined by

$$f(x) = 100(x_2 - x_1^2)^2 + (1 - x_1)^2.$$

This function is not convex in $\mathbb{R}^2$ with the usual metric. But we show that f is convex in the Riemannian manifold $(\mathbb{R}^2, g)$, where the metric g is given by

$$g(x) = \begin{pmatrix} 4x_1^2 + 1 & -2x_1 \\ -2x_1 & 1 \end{pmatrix}, \quad x = (x_1, x_2).$$

Here $g_{11} = 4x_1^2+1, g_{12} = -2x_1, g_{21} = -2x_1, g_{22} = 1$.We know that g^{ij} are the entries of the inverse matrix $[g_{ij}]$. Hence, $g^{11} = 1, g^{12} = 2x_1, g^{21} = 2x_1, g^{22} = 4x_1^2 + 1$ and the Christoffel symbols are $\Gamma^1_{11} = \Gamma^1_{12} = \Gamma^1_{22} = \Gamma^2_{12} = \Gamma^2_{22} = 0$