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A VISCOELASTIC PLATE EQUATION WITH A VERY GENERAL KERNEL

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Abstract. The stabilization of the following problem:

$$u_{tt} - \sigma \Delta u_{tt} + \Delta^2 u - \int_0^t k(t-s) \Delta^2 u(s) ds = 0, \qquad x \in \Omega, \quad t > 0,$$

is investigated under very general assumption on the relaxation function k. With this general assumption, $k'(t) \leq -\gamma(t)\Psi(k(t))$, we establish general and optimal decay rate results from which we recover the optimal rates when $\Psi(s) = s^p$ and p covers the full admissible range [1, 2). Our results improve and generalize many earlier results in the literature.

Keywords. Viscoelastic, Plate equation, Optimal decay, Kernels, Convexity.

AMS (MOS) subject classification: 35B40; 74D99; 93D15; 93D20.

1 Introduction

Viscoelastic plate equations have been studied by many authors and several stability results have been established. For example, Rivera et al. [36] studied the following initial-boundary problem for viscoelastic plate equation,

$$u_{tt} - \sigma \Delta u_{tt} + \Delta^2 u + \int_0^t k(t-s)\Delta^2 u(s)ds = 0. \text{ in } \Omega \times \mathbb{R}^+$$
(1)

with initial and dynamical boundary conditions and a relaxation function k satisfies the following conditions

$$-c_0 k(t) \le k'(t) \le -c_1 k(t), \qquad 0 \le k''(t) \le c_2 k(t), \tag{2}$$

for some positive constant c_i , i = 0, 1, 2 and the constant $\sigma = \frac{h^2}{12}$, where h is the uniform thickness of the plate. They demonstrated that the sum of the first and second-order energies decay exponentially (polynomially) when the kernels do so. More precisely, if the relaxation function k satisfies the following condition

$$k(t) \le -c_0 k^{1+\frac{1}{p}}(t); \quad k, k^{1+\frac{1}{p}} \in \mathbb{L}^1(\mathbb{R}) \quad p > 2,$$
 (3)