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AN ITERATIVE METHOD TO SOLVE A NONLINEAR THREE-TIME-SCALE DISCRETE SYSTEM

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Abstract. The paper considers a two point boundary value problem for a three-time-scale state model arising in the open-loop optimal control of singularly perturbed discrete systems. An algorithm for constructing asymptotic solutions is described.

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1 Introduction

Discrete-time systems with singular perturbation structures have been extensively studied in engineering and control theory and have many applications in circuit theory, economics, and other fields. Their fundamental characteristic is the lack of a single model, the high dimensionality and the multiple time-scales. Details of recent developments in the theory of singularly disturbed systems and their applications are given in [2, 5, 7, 21]. In this paper, we study a two-point boundary value problem (TPBVP) of a class of nonlinear discrete singularly perturbed systems with boundary layers occurring at the initial and final values. Such TPBVPs arise in the open-loop formulation of control problems (see, e.g. [4]). We consider the system

$$\begin{cases} x(t+1) = f\left(x(t), \varepsilon y(t), z(t), \varepsilon, t\right), \\ y(t+1) = g\left(x(t), \varepsilon y(t), z(t), \varepsilon, t\right), \\ \varepsilon z(t+1) = h\left(x(t), \varepsilon y(t), z(t), \varepsilon, t\right), \end{cases} \qquad t \in I_{N-1}, \tag{1.1}$$

with the boundary conditions

$$x(t=0) = \alpha_1(\varepsilon), \quad y(t=0) = \alpha_2(\varepsilon), \quad z(t=N) = \beta(\varepsilon), \tag{1.2}$$

supposed to satisfy for $|\varepsilon| < \delta \leq 1$,

$$\alpha_j(\varepsilon) = \sum_{i=0}^{n} \varepsilon^i \alpha_j^{(i)} + o(\varepsilon^n), \; j = 1, 2, \quad \beta(\varepsilon) = \sum_{i=0}^{n} \varepsilon^i \beta^{(i)} + o(\varepsilon^n), \tag{1.3}$$