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DISCRETE FRACTIONAL LYAPUNOV-TYPE INEQUALITIES IN NABLA SENSE

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Abstract. In this article we establish a few Lyapunov-type inequalities for two-point nabla fractional boundary value problems associated with conjugate, left focal and right focal boundary conditions. To illustrate the applicability of established results, we examine disconjugacy and disfocality as a nonexistence criterion for nontrivial solutions and estimate lower bound for eigenvalues of the corresponding nabla fractional eigenvalue problem.

Keywords. Fractional order, backward (nabla) difference, boundary value problem, Green's function, Lyapunov inequality, disconjugacy, disfocality

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1 Introduction

In 1907, Lyapunov [34] proved the following result which provides a necessary condition for the existence of a nontrivial solution of Hill's equation associated with Dirichlet boundary conditions.

Theorem 1.1. *[34] If the boundary value problem*

$$\begin{cases} y''(t) + p(t)y(t) = 0, \quad a < t < b, \\ y(a) = 0, \quad y(b) = 0, \end{cases} \tag{1.1}$$

has a nontrivial solution, where $p : [a, b] \to \mathbb{R}$ *is a continuous function, then*

$$\int_a^b |p(s)| ds > \frac{4}{(b-a)}. \tag{1.2}$$

The inequality (1.2), known as Lyapunov inequality, has several applications in various problems related to differential equations, including oscillation theory, asymptotic theory, eigenvalue problems, disconjugacy etc. Due to its importance, the Lyapunov inequality has been generalized in many forms. For more details on Lyapunov-type inequalities and their applications, we refer [11, 39, 41, 45, 49] and the references therein.