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ON A PRODUCT-TYPE DIFFERENCE EQUATION OF HIGHER-ORDER

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Abstract.
This work examines the periodicity, stability and form of solutions of the product-type difference equation

$$x_{n+1} = \frac{x_n\, x_{n-2}\, \cdots\, x_{n-2k}}{x_{n-1}\, x_{n-3}\, \cdots\, x_{n-2k-1}}, \quad n = 0, 1, 2, \dots,$$

with initials $\{x_n\}_{-2k-1}^{0} \in \mathbb{R}$ and k is some fixed non-negative integer. This study also provides some sort of generalization of the results exhibited by the second author in [14].

Keywords. Product-type difference equation, solution form, periodicity, stability.

AMS (MOS) subject classification: 39A10, 40A05.

1 Introduction

A difference equation is a particular example of what is known as a recurrence equation. A classical example of this type of equation is the widely known recurrence equation $A_{n+1} = A_n + A_{n-1}$ with initials $A_0 = 0$ and $A_1 = 1$. Such type of recurrence equation can be classified as a linear difference equation.Meanwhile, a recurrence equation $y_{n+1} = h(y_n, y_{n-1}, \dots, y_{n-k})$ such that h defined as ratios of polynomials $P(y_n, y_{n-1}, \dots, y_{n-k})$ and $Q(y_n, y_{n-1}, \dots, y_{n-k})$ is classified as a nonlinear difference equation. A particular example of this type of equation is the so-called Riccati difference equation:

$$y_{n+1} = \frac{by_n + a}{c + dy_n}, \quad n = 0, 1, \dots, \tag{I.1}$$

which has been investigated recently by some authors, see, for instance, Brand [5] and Papaschinopoulos and Papadopoulos [24]. Several recent investigations focusing on some instances of the Riccati difference equation