

## EXISTENCE AND UNIQUENESS OF SOLUTIONS FOR MULTI-TERM FRACTIONAL LANGEVIN EQUATION WITH BOUNDARY CONDITIONS

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**Abstract.** We investigate the existence and uniqueness of solutions for a new class of boundary value problems of multi-term Langevin equation involving Caputo fractional derivatives of different orders. The modern tools of functional analysis are applied to establish the main results, which are well-supported with examples.

**Keywords.** Caputo fractional derivative, Fractional Langevin equation, existence, fixed point

**AMS (MOS) subject classification:** 34A08; 34B15; 26A33

## 1 Introduction

Fractional order differential equations received overwhelming attention of many researchers as these equations extensively appear in the mathematical modeling of several scientific and technical phenomena. Examples include physics, biology, chemistry, control theory, electrical circuits, wave propagation, blood flow phenomena, signal and image processing, etc. For further details, see [1]-[5].

Langevin equation, formulated in terms of integer-order derivatives by Langevin [6] in 1908, is a well-known equation of mathematical physics, which is used to describe the evolution of physical phenomena, such as Brownian motion, in fluctuating environments.

Langevin equation is also known as a stochastic differential equation as it is related to the fast motion of microscopic variables of the dynamical systems. However, the failure of classical Langevin equation to describe the complex systems led to its several generalizations, which successfully modeled the physical phenomena in disordered regions [7], anomalous diffusion processes in complex and viscoelastic environment [8, 9], etc. Among these