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ASYMPTOTIC BEHAVIOR AND PERIODIC SOLUTIONS TO A FIRST ORDER EXPANSIVE TYPE DIFFERENCE EQUATION

B. Djafari Rouhani[1] and M. Rahimi Piranfar[1,2]

[1]Department of Mathematical Sciences
University of Texas at El Paso, 500 W. University Ave., El Paso, TX 79968, USA

[2]Department of Mathematics
Institute for Advanced Studies in Basic Sciences, P.O. Box 45195-1159, Zanjan, Iran

Abstract. We consider the following difference equation

$$\frac{u_{n+1} - u_n}{\lambda_n} \in Au_n, \tag{FDE}$$

where A is a maximal monotone operator. We study the asymptotic behavior of the solutions to (FDE). We also prove that periodic solutions to (FDE) exist when A is a single-valued and maximal strongly monotone operator. Moreover if periodic solutions to (FDE) exist, then all bounded solutions to (FDE) are periodic. Our work is motivated by [5, 6].

Keywords. almost expansive sequence, asymptotic behavior, maximal monotone operator, difference equation, periodic solution

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1 Introduction and preliminaries

Let H be a real Hilbert space endowed with scalar product $\langle \cdot, \cdot \rangle$, induced norm $\| \cdot \|$ and identity operator I. An operator $A : D(A) \subset H \to H$ (possibly multivalued) is called monotone (respectively strongly monotone) if $\langle y_2 - y_1, x_2 - x_1 \rangle \geq 0$, (respectively $\langle y_2 - y_1, x_2 - x_1 \rangle \geq \alpha \|x_2 - x_1\|^2$ for some $\alpha > 0$) for all $x_i \in D(A)$, $y_i \in A(x_i)$, $i = 1, 2$. A monotone operator A is maximal if $R(I + A) = H$. Asymptotic behavior of solutions to the autonomous dissipative system

$$\begin{cases} -\dot{u}(t) \in Au(t), \\ u(0) = u_0 \in D(A), \end{cases} \tag{1.1}$$

where A is a maximal monotone operator in H with a nonempty zero set, was investigated by Baillon and Brzis [1]. By introducing the notion of