

THE FORM OF THE SOLUTIONS AND BEHAVIOR OF SOME NONLINEAR DIFFERENCE EQUATIONS

E. M. Elsayed^{1,2} and N. H. Alotaibi¹

¹Mathematics Department, Faculty of Science,

King Abdulaziz University, P. O. Box 80203,
Jeddah 21589, Saudi Arabia.

²Department of Mathematics, Faculty of Science,
Mansoura University, Mansoura 35516, Egypt.

E-mails: emmelsayed@yahoo.com, faris.kau@hotmail.com,
n.h.ob@hotmail.com.

Abstract. In this paper, we study some qualitative behavior such as boundedness, local stability and global attractor of the solutions of the following difference equations

$$x_{n+1} = \frac{ax_n x_{n-3}}{bx_n + cx_{n-3}}, \quad x_{n+1} = \frac{ax_n x_{n-3}}{bx_n - cx_{n-3}},$$

where the initial conditions x_{-3} , x_{-2} , x_{-1} and x_0 are arbitrary positive real numbers, and a , b , c are positive constants. Also, we obtain the form of the solutions of the special case of these equations.

Keywords. stability, periodicity, boundedness, recursive sequence, difference equation.

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1 Introduction

The theory of difference equations has been recently studied all over the world. Take, for example, Abo-Zeid [2] analyzed the global behavior of all solutions of the difference equation

$$x_{n+1} = \frac{x_n x_{n-1}}{ax_n + bx_{n-1}}.$$

Cinar [5] obtained the positive solutions of the difference equation

$$x_{n+1} = \frac{ax_{n-1}}{1 + bx_n x_{n-1}}.$$

Aloqeili [4] investigated the solutions of the following difference equation

$$x_{n+1} = \frac{x_{n-1}}{a - x_n x_{n-1}}.$$