

## ECONOMIC GROWTH AND POPULATION MODELS: A DISCRETE TIME ANALYSIS

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**Abstract.** This paper studies an extension of the Mankiw-Romer-Weil growth model in discrete time by departing from the standard assumption of constant population growth rate. More concretely, this rate is assumed to be decreasing over time and a general population growth law verifying this characteristic is introduced. In this setup, the model can be represented by a three dimensional dynamical system which admits a unique solution for any initial condition. It is shown that there is a unique nontrivial equilibrium which is a global attractor. In addition, the speed of convergence to the steady state is characterized, showing that in this framework this velocity is lower than in the original model.

**keywords:** Mankiw-Romer-Weil economic growth model; discrete time; decreasing population growth rate; speed of convergence.

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### 1 Introduction

An obligatory reference in the studies on economic growth and its determinants, particularly in the empirical ones, is the model developed by Mankiw, Romer and Weil [33]-also known as the Solow model extended with human capital. The model assumes that labor force (associated with the size of the population) grows at a constant rate  $n > 0$ . This assumption, normally used in the classic growth models (Solow [40], Ramsey [36], Cass [16], Koopmans [30] among others) implies that the population grows exponentially, i.e. in discrete time if the initial population is  $L_0$ , the population at time  $t$  is  $L_t = L_0(1 + n)^t$ . Assuming that population growing exponentially implies that there is no limit to the size of the population (it tends to infinity as  $t$  tends to infinity). This assumption is clearly not sustainable, as it does not fit with recent empirical data of the last hundred years [42].

The exponential model conforms the dynamics of the population in initial periods, but it is unable to reflect the fall in the rate of growth due to-