ON THE DYNAMICS OF A NEW NONLINEAR RATIONAL DIFFERENCE EQUATION

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Abstract. We investigate in this article, the periodicity, the boundedness and the global
stability of the positive solutions of the following new nonlinear rational difference equation

\[ x_{n+1} = \frac{x_n x_{n-k} x_{n-l}}{(a x_n + b x_{n-k} + c x_{n-l})}, \quad n = 0, 1, 2, \ldots \]

where the parameters \(a, b, c\) are positive integers, such that \(k < l\). The initial conditions \(x_{-1}, x_{-1+1}, \ldots, x_{-k}, x_{-k+1}, \ldots, x_{-1}, x_0\) are arbitrary positive real numbers, while \(k, l\) are positive integers, such that \(k < l\). Recently there has been a great interest
in studying rational and nonrational difference equations \([1 - 37]\). Some of
the results recently obtained in this field can be applied in studying some
mathematical biology models, population, dynamics, ecology, physics, economy, technics, sociology etc. The objective of this paper is to study the global
attractivity, the boundedness character and the periodic nature of Eq.(1).
To this end, we recall some notations and results which will be useful in our
investigation (see \([1 - 37]\)).

Let \(I\) be some interval of real numbers and let \(F : I^{k+1} \rightarrow I\), be a
continuously differentiable function. Then for every set of initial conditions
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