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ON THE DYNAMICS OF A NEW NONLINEAR RATIONAL DIFFERENCE EQUATION

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Abstract. We investigate in this article, the periodicity, the boundedness and the global stability of the positive solutions of the following new nonlinear rational difference equation

$$x_{n+1} = \frac{x_n x_{n-k} x_{n-l}}{(ax_n + bx_{n-k} + cx_{n-l})}, \qquad n = 0, 1, 2, ...$$

where the parameters $a, b, c \in (0, \infty)$,while k, l are positive integers, such that $k < l$. The initial conditions $x_{-l},\ x_{-l+1}, ...,\ x_{-k},\ x_{-k+1},\ ...,\ x_{-1},\ x_0 \in (0, \infty)$. Some numerical examples are given. words.

Keywords. Difference equations; Rational difference equations; Global stability; locally asymptotic stable; prime period two solution.

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1 Introduction

In this paper, we consider the following new nonlinear rational difference equation

$$x_{n+1} = \frac{x_n x_{n-k} x_{n-l}}{(ax_n + bx_{n-k} + cx_{n-l})}, \qquad n = 0, 1, 2, ... \tag{1}$$

where the parameters a, b, c and the initial conditions $x_{-l},\ x_{-l+1},\ ...,\ x_{-k},\ x_{-k+1},\ ...,\ x_{-1},\ x_0$ are arbitrary positive real numbers, while k and l are positive integers, such that $k < l$. Recently there has been a great interest in studying rational and nonrational difference equations $[1-37]$. Some of the results recently obtained in this field can be applied in studying some mathematical biology models, population, dynamics, ecology, physics, economy, technics, sociology etc.The objective of this paper is to study the global attractivity, the boundedness character and the periodic nature of $Eq.(1)$. To this end, we recall some notations and results which will be useful in our investigation (see $[1-37]$):

Let I be some interval of real numbers and let $F : I^{k+1} \to I$, be a continuously differentiable function. Then for every set of initial conditions $x_{-k}, x_{-k+1}, ..., x_{-1}, x_0 \in I$, the difference equation