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THE UNILATERAL SHIFT IDE

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Abstract. This paper provides a proof that the pointwise difference between the unilateral shift IDE and $2x\,\mathrm{mod}(1)$ can be made arbitrarily small as the step size parameter, h, goes to zero.

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1 Introduction

The main result of this paper is Theorem 1 which states that the time one map of the $x-$component of the unilateral shift (ULS) IDE converges pointwise to $2x\,\mathrm{mod}(1)$, an unilateral shift (ULS) on two symbols, as $h \to 0$. This result is a step toward resolving The Hirsch Conjecture [1]. The foundations for the results presented here are in [2]

The proof of Theorem 1 is presented in two forms: (1) an informal statement that makes minimal use of definitions and formulas in Sec. 2; and (2) a formal proof in Sec. 3. The difference between these two proofs is that the informal proof is presented in non-computational language with extensive commentary to aid in the understanding of the proof. The formal proof is presented in terms of formal definitions, lemmas and equations.

A formal statement of the main result of this paper is Theorem 1. The relevant definitions are presented in Sec. 3.

Theorem 1 *Consider* $\hat{\mathbf{T}}_h$ *with the initial conditions*

$$\mathscr{X}_0 = (\mathbf{e}_2,\ 1.0,\ 0.5 \cdot \mathbf{e}_1, x_0\,\mathbf{e}_1)$$

where $x_0 \in (0,1)$ *and has positive algorithmic complexity* [3], [4].

Then

(1) *if* $x_0 < 0.5$, $\mathbf{T}_h^M(x_0\mathbf{e}_1) = 2\,x_0\mathbf{e}_1$.

(2) *If* $x_0 \in (0.5, 1.0)$, *then* $\mathbf{T}_h^M(x_0\,\mathbf{e}_1) = (2^{1-h}\,x_0 - 1)\,\mathbf{e}_1$.

(3) *If* $x_0 \in (0.5, 1.0)$, *then* $\mathbf{T}_h^M(x_0\,\mathbf{e}_1) = 2\,x_0\,\mathrm{mod}(1) + \varepsilon$

(4) $\varepsilon < 2\,\|1 - 2^{-h}\|$