Dynamics of Continuous, Discrete and Impulsive Systems
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IDE THEORY AND CHAOS

Ray Brown[1]

[1]EEASI Corporation
Houston, Texas 77057

Abstract. This paper *demonstrates* that a sufficient condition to determine whether an ODE or an IDE can be determined to have chaotic solutions based on its *form* is that it is possible to identify the elementary dynamics and their boundaries in the form of the ODE or IDE.

A central conclusion that is demonstrated by the examples in this paper is that it is the boundary or interface between the elementary IDEs that determines whether their combined dynamics will produce chaos. A very small perturbation of the boundary can produce a periodic IDE.

Keywords. chaos, natural science, complexity, nonlinear dynamics, dynamical synthesis.

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1 Introduction

This paper presents the diverse forms of IDEs which are chaotic. examples of the Forms of Chaotic systems to be used to address the Hirsch Conjecture. These examples will also be used to guide the development of theorems in a later paper related to the Hirsch Conjecture.

The Importance of Boundary Conditions. There are two components needed in the formation of systems that produce chaos. (1) The dynamical processes that appear in the system; (2) the boundary between those processes. For example, in the Hénon IDE there is an elliptic fixed point and an hyperbolic fixed point and there is a boundary where the hyperbolic dynamics end and the elliptic dynamics begin. For a simplified example projected onto a two-dimensional plane, see Fig.1

Figure 1 is generally the form of systems that produce chaos. The Chua double scroll is a good example. A second form is where there is no explicit hyperbolic region but there is a local region where two dynamics converge to form an hyperbolic region. This is illustrated in Fig. 2

In Fig. 3, there is a hyperbolic fixed point at the origin and an elliptic fixed point at $(0.4, 0.3)$. The boundary is $y = 1 - x$. Figure 3 provides an example of the fusion of two elementary IDEs, see Sec. 2, an exponential and elliptic IDE. The figure also illustrates the fact that no mater which IDEs one chooses, a two-dimensional differentiable fusion cannot produce chaos as a consequence of the Poincaré- Bendixson