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MATHEMATICAL FOUNDATIONS OF IDE THEORY

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Abstract. This paper will present the mathematical foundations of IDE Theory. The topics will include fundamental theorems; IDEs from ODEs; perturbations of IDEs; combing IDEs; embedding a general diffeomorphism in an IDE as the time one map; numerical analysis of IDEs

Keywords. chaos, algebra, complexity, dynamical systems, dynamical synthesis.

AMS (MOS) subject classification: 37D45.

1 Introduction

The topics that will be covered in this paper are IDE axioms; the fundamental theorems of the standard IDE; IDEs that are related to various ODEs; perturbations of the standard IDE; time-dependent IDEs; methods of combining IDEs to form more complex IDEs; the embedding of an arbitrary diffeomorphism in an IDE; and numerically fitting an IDE to a set of measurements.

In the following, all results will apply to complex, $\mathbb{C}^n$, and real vector spaces, $\mathbb{R}^b$ unless otherwise stated. All results can be obtained for complex space by considering the real and imaginary parts separately or by viewing n-dimensional complex space as 2n-dimensional real space with appropriate adjustments.

2 Fundamental Theorems

This section presents the fundamental theorems on which the theory of IDEs is based. This will require stating axioms and some preliminary results. The axioms of IDEs will be slightly generalized to include more possibilities. The key feature of an IDE is that from one step to another the value of the IDE does not change "dramatically" as does occur in ordinary finite difference equations or diffeomorphisms. In particular, any change should depend on a step size parameter, h, that can be made as small as needed for each application. This requirement assures that the orbit of an IDE closely approximates a continuous curve in space and when related to an ODE, closely follows the dynamics of the solution of an ODE in a morphological sense.