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A NEW MATHEMATICAL DISCIPLINE

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Abstract. This paper introduces a new mathematical discipline, Infinitesimal Diffeomorphism Equations (IDE), which is a subject within the field of Dynamical Systems. In contrast to the theory of Global Dynamics, IDE Theory is the study of Local Dynamics. The role of local dynamics and its significance to global dynamics, chaos and the modeling of complex systems will be the subject of this Issue.

Keywords. Chaos, natural science, complexity, dynamical systems, dynamical synthesis.

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1 What are Infinitesimal Diffeomorphism Equations (IDE)?

Infinitesimal Diffeomorphism Equations (IDE) arose as an means of addressing the Hirsch Conjecture:

> *"A major challenge to mathematicians is to determine which dynamical systems are chaotic and which are not. Ideally one should be able to tell from the form of the differential equation"* – Morris W. Hirsch, 1985 [7], page 192.

Let $\mathbf{A}$ be an $n \times n$ matrix of constants in $\mathbb{R}$ and $\mathbf{X}$ and n-dimensional vector in $\mathbb{R}^n$. Consider the liner vector ODE

$$\dot{\mathbf{X}} = \mathbf{A}\,\mathbf{X} \quad \mathbf{X}(0) = \mathbf{X}_0$$

The global solution is

$$\mathbf{X}(t) = \exp(\mathbf{A}\,t)\,\mathbf{X}_0$$

The local solution is

$$\mathbf{T}_h(\mathbf{X}) = \exp(h\,\mathbf{A})\,\mathbf{X}_0$$

where h is a small parameter which may be thought of as a step size. It is convenient to think of $\mathbf{T}_h = \exp(h\,\mathbf{A})$, an exponential operator, as an operator on $\mathbb{R}^n$. IDE Theory is an abstraction of the properties of the exponential operator on $\mathbb{R}^n$.