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EXISTENCE OF ANTI-PERIODIC SOLUTIONS FOR A SECOND ORDER NEUTRAL FUNCTIONAL DIFFERENTIAL EQUATION VIA LEARY-SCHAUDER'S FIXED POINT THEOREM

Ahlème Bouakkaz[1], Abdelouaheb Ardjouni[2] and Ahcene Djoudi[3]

[1]Laboratoire LAMAHIS, Département de Mathématiques
Université 20 Août 1955 -Skikda 21000, Algérie

[2]Department of Mathematics and Informatics
University of Souk Ahras, P.O. Box 1553, Souk Ahras, 41000, Algeria

[3]Department of Mathematics
University of Annaba, P.O. Box 12, Annaba 23000, Algeria

Abstract. In this work, by the Leray-Schauder fixed point theorem, some new results on existence of anti-periodic solutions for a class of second order neutral differential equation are obtained. we constructed an example to show the application of our main results.

Keywords. Periodic solutions, nonlinear differential equations, fixed point theorem, Green's function.

AMS (MOS) subject classification: Primary 34K13, 34A34; Secondary 34K30, 34L30.

1 Introduction

Consider the following second order neutral differential equation which can be considered as a neutral Rayleigh-type equation

$$\begin{aligned}&(x(t)-bx(t-r(t)))''+f(t,x'(t),x'(t-r(t)))\\&+g(t,x(t),x(t-r(t))) \;=\; e(t),\end{aligned}\tag{1}$$

where $|b|<1$, $e:\mathbb{R}\longrightarrow\mathbb{R}$ and $f,g:\mathbb{R}\times\mathbb{R}\times\mathbb{R}\longrightarrow\mathbb{R}$ are T-periodic continuous functions in their first argument, $r\in\mathcal{C}^2(\mathbb{R},\mathbb{R})$, e and r are T-periodic functions with $T>0$ is a constant. This type of equations have been received a lot of attentions since they have widespread applications in many areas of applied sciences as physics, mechanics, engineering technique fields, biology... (see for example [5, 6, 11, 13, 18, 25]).

It is worth mentioning that the study of anti periodic problems began with the pioneering works of H. Okochi and A. Haraux. In fact, H. Okochi [21, 22, 23] was the first to introduce the concept of anti-periodic solutions for nonlinear evolution equations in a real Hilbert space and via Schauder's fixed point theorem some results on existence and uniqueness of anti-periodic solutions were obtained by Haraux in its monograph [16].