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## ASYMPTOTIC BEHAVIOUR AND SQUARE INTEGRABILITY OF THIRD ORDER NONLINEAR VECTOR DELAY DIFFERENTIAL EQUATION

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**Abstract.** In this paper, we consider a class of non-linear vector differential equations of third order with delay and we study the asymptotic stability, boundedness, ultimately boundedness and square integrability of solutions. The technique of proofs involves defining an appropriate Lyapunov functional. The obtained results include and improve the results in the literature.

**Keywords.** Boundedness, stability, square integrability, Lyapunov functional, third-order delay vector differential equations.

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## 1 Introduction

During the last years, there has been a strong interest in the study of the Boundedness and stability theory of third-order delay differential equations. As is well known, the third-order differential equations are derived from many different areas of applied mathematics and physics, for instance, deflection of buckling beam with a fixed or variable cross-section, three-layer beam, electromagnetic waves, gravity-driven flows, etc.

For some works on the stability and boundedness in scalar ordinary and functional differential equations of third order without and with delay, we refer the interested reader to the papers of Fatmi and Remili [11], Graef et al.[12, 13], Oudjedi et al.[19], Remili et al. [23]-[30] and Tunç [33]-[36].

In 1985, Abou El Ala [1] gave sufficient conditions that ensure that all solutions of real vector differential equations of the form

$$X''' + F(X, X')X'' + G(X') + H(X) = P(t, X, X', X''),$$
(1.1)

are ultimately bounded. Afterward, in 2006 Tunç and Tunç [39] also proved some results on the asymptotic stability and the boundedness of solutions