

STABILITY OF INVARIANT MANIFOLDS OF CERTAIN CLASSES OF SET DIFFERENTIAL EQUATIONS

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Abstract. New approaches of studying the stability of invariant manifolds of Set Differential Equations (SDEs) in the space $\text{conv } \mathbb{R}^2$ are proposed based on Lyapunov's direct method, stability in terms of two measures, and the theory of mixed volumes. For pseudo-linear set differential equations, we establish existence of an invariant manifold of orbits of solutions in the quotient space of convex compact sets by the homothety group. We also obtain some stability conditions for such an invariant manifold.

Keywords. Set Differential Equations, invariant manifold, Brunn–Minkowski inequality, stability in terms of two measures, Lyapunov's direct method, comparison method, mixed area.

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1 Introduction

Differentiability of a multivalued mapping with values in the space of convex compact sets, $\text{conv } \mathbb{R}^n$, was introduced in [8] and the corresponding derivative is called the Hukuhara derivative. Originally, Set Differential Equations (SDEs) with a Hukuhara derivative were studied in [5]. In recent years the qualitative and stability theory of SDEs has received a lot of attention. In particular, we refer to the monograph [9], the papers [7, 12, 14] and the references contained therein. In [9], some results on the general theory of SDEs have been summarized and theorems generalizing Lyapunov's direct method and the comparison method have been obtained.

However, in the review of the monograph [9] it is indicated that in the monograph "... the definition of stability and asymptotic stability are not explicitly stated. The latter is a nontrivial issue, since the diameter of a solution is nondecreasing."

The following example (see [9]) illustrates the difficulties that arise in the study of stability of solutions of SDEs. This also highlights the differences in the dynamic properties of solutions of ordinary differential equations and their SDE analogues.