

BIFURCATIONS IN A CHAOTIC DUOPOLY GAME WITH A LOGARITHMIC DEMAND FUNCTION

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Abstract. In this paper we study the dynamics behaviors of a Cournot type model with homogenous goods, log-linear price function and quadratic cost functions. We show that the model can only have three fixed points. A local stability analysis is performed for these points. Moreover, we show that the model undergoes a period-doubling bifurcation points where a two cycles of period 2 are born. The existence of period-doubling bifurcations on the 2-cycles curves indicate the existence of cycles of higher periods as well. A numerical stability analysis of the 2-cycles is performed. It shown that one of the 2-cycles always unstable while the other is stable in the region bounded by two successive period-doubling points. Finally, the largest Lyapunov exponent confirms the predominance of a chaotic attractor for the parameter values greater than 0.8.

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1 Introduction

Over the last two decades much research has analyzed the complex dynamics behaviors appear in oligopolistic markets models both under quantity and price competition. The oligopolistic market structure set between the extremes of perfect and monopoly competition, see for example [18, 13, 2]. In oligopolistic setting, the market firms (players) have a strong but incomplete influence on market prices for the goods they sell. This add more complexity not seen in perfect and monopoly competition where players must consider not only their own production but also incorporate the reactions of other market players.

Cournot [7] introduces the first mathematical model of duopoly competition, *i.e.*, oligopoly with two players. His model examine two oligopolistic sellers competing with respect to the quantity of good they will produce. The goods are assumed to be homogenous, meaning that there is no way for buyers to differentiate between goods. Thus the price at which goods are sold is completely determined by the quantity of goods produced and