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MATHEMATICAL MODELLING AND NONLINEAR CONTROL OF A REAR-WHEEL-DRIVE VEHICLE BY USING THE NEWTON-EULER EQUATIONS - PART 1 [1]

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Abstract. This is Part 1 of a two part series of works dealing with the mathematical modelling and nonlinear control of a rear-wheel-drive vehicle by using the Newton-Euler equations and the d'Alembert principle. The vehicle is controlled by two applied torques. First, there is a steering wheel system torque steering the front wheels via an internal steering system mechanism leading to holonomic velocity constraints. Second, there is a drive system torque driving the rear wheels via a differential gearbox and side shafts leading to holonomic velocity constraints. It is assumed that all four wheels of the vehicle roll perfectly resulting in nonholonomic velocity constraints. It turns out that the vehicle systems lead to a set of holonomic and nonholonomic velocity constraints that are not independent. In this work, the constraints are not converted to a set of independent velocity constraints. The original form and structure of the constraints are preserved. In Part 2, the d'Alembert principle is applied in order to extend the Newton-Euler equations for the case where the velocity constraints may not be independent. The developed methodology is applied here in order to derive the kinematic and dynamic models of the vehicle by using all the velocity constraints. In addition, a nonlinear feedback control law is derived for the steering wheel system torque and the drive system torque such that the steering wheel system turning angle and the drive system rotational velocity asymptotically track specified reference trajectories, respectively. The constrained motion of the controlled vehicle dynamic model is computed and used to obtain the vector of generalized constraint forces. Subsequently the vector of Lagrange multipliers is obtained by computing the Moore-Penrose generalized inverse of the velocity constraints matrix.

Keywords. Nonlinear control, Newton-Euler equations, d'Alembert principle, State space form, Generalized applied forces, Generalized constraint forces, Independent and not independent velocity constraints, Velocity constraints matrix, Kinematic model, Reduced dynamic model, Lagrange multipliers, Moore-Penrose generalized inverse, Rear-wheel-drive vehicle, Differential gearbox, Front wheel steering mechanism, Instantaneous center of rotation, Geometric constraints, Nonholonomic and holonomic velocity constraints.

[1] Gratefully dedicated to the memory of our unforgettable and beloved mother Angeliki Frangos (*Αγγελική Φράγκου*), 1926-2016. Constantinos Frangos, Evangelia Frangos.