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# ON THE CAUSALITY OF SINGULAR SYSTEMS OF DIFFERENCE EQUATIONS OF FIRST AND FRACTIONAL ORDER

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**Abstract:** We consider two type of systems, a linear singular discrete time system and a linear singular fractional discrete time system whose coefficients are square constant matrices. By assuming that the input vector changes only at equally space sampling instants we investigate and provide properties for causality between state and inputs and causality between output and inputs.

**Keywords** : causality, linear, fractional, singular, system.

## 1 Introduction

Many authors have studied generalized discrete & continous time systems, see [1–22], and their applications, see [23–32]. Many of these results have already been extended to systems of differential & difference equations with fractional operators, see [33–42].

If we define $\mathbb{N}_\alpha$ by $\mathbb{N}_\alpha = \{\alpha, \alpha+1, \alpha+2, ...\}$, $\alpha$ integer, and $n$ such that $0 < n < 1$ or $1 < n < 2$, then the nabla fractional operator in the case of Riemann-Liouville fractional difference of $n$-th order for any $Y_k : \mathbb{N}_a \to \mathbb{R}^m$ is defined by

$$\nabla_\alpha^{-n} Y_k = \frac{1}{\Gamma(n)} \sum_{j=\alpha}^{k} (k-j+1)^{\overline{n-1}} Y_j.$$

We denote $\mathbb{R}^{m\times 1}$ with $\mathbb{R}^m$. Where the raising power function is defined by

$$k^{\bar{\alpha}} = \frac{\Gamma(k+\alpha)}{\Gamma(k)}.$$

We consider the singular discrete time system of the form

$$FY_{k+1} = GY_k + V_k, \quad k = 1, 2, ..., \tag{1}$$

the singular fractional discrete time system of the form

$$F\nabla_0^n Y_k = GY_k + V_k, \quad k = 1, 2, ..., \tag{2}$$