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# STABILITY AND HOPF BIFURCATION ANALYSIS FOR A PREDATOR-PREY MODEL WITH TWO DELAYS

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**Abstract.** In this research, we consider a Predator-prey mathematical model with two delays. The stability of the unique positive equilibrium point is investigated. The Hopf bifurcation analysis is performed and the conditions for Hopf bifurcation are established. The bifurcation diagram is described in terms of the delays and the normal forms are derived using the normal form theory. Numerical simulations are provided to illustrate the theoretical results.

**Keywords.** Predator-prey model, Delay differential equations, Hopf bifurcation, Stability, normal forms.

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## 1 Introduction

Since the delayed predator-prey models were introduced and studied by Volterra [18, 19] in 1920s, this types of systems, called generally nowadays Lotka-Volterra Predator-prey systems, have been modified, studied extensively in the studies of population dynamics. Delays can be either distributed or discrete(see, for example, monographs by [5, 12, 14, 26], etc.), and can be inter-specific, predator specific, or prey specific(see [2, 5, 8, 10, 13, 15, 17] and references therein). For a detailed discussion for delayed predator-prey systems, we refer the readers to Cushing [5], Kuang [10], MacDonald [12] and a survey paper by Ruan [16]. Delayed predator-prey systems have shown a rich dynamical behaviors such as Hopf bifurcation, Bogdanov-Takens bifurcation, and even chaos.

A predator-prey system with delayed inter-specific interaction generally takes the following form

$$\begin{cases} x'(t) = x(t)[r_1 - a_1 x(t) - a_2 y(t-\tau)], \\ y'(t) = y(t)[-r_2 + a_3 x(t-\rho) - a_4 y(t)], \end{cases} \tag{1}$$