

TRANSITIVITY OF FLOWS WITH LIMIT SHADOWING

Manseob Lee¹ and Junmi Park²

¹Department of Mathematics
Mokwon University, Daejeon, 302-729, Korea

²Department of Mathematics
Chungnam National University, Daejeon, 305-764, Korea

E-mail: lmsds@mokwon.ac.kr, pjmds@cnu.ac.kr

Abstract. We show that if a flow has the limit shadowing property then it is chain transitive. Furthermore, it is topologically transitive.

2010 Mathematical Subject Classification: 37C50

Key words and phrases: shadowing, limit shadowing, chain transitive, topologically transitive, Lyapunov stable, minimal.

1 Introduction

Let (X, d) be a compact metric space with metric d , and let $f : X \rightarrow X$ be a continuous map. The various pseudo orbit tracing properties (shadowing, average shadowing, asymptotic average shadowing, ergodic shadowing etc.) are very important topics of the dynamical systems. For instance, Li has shown in [13] that if f is chain transitive with shadowing property then it is topologically ergodic. In [14], Park and Zhang proved that if a continuous surjective map f has the average shadowing property then it is chain transitive. Gu [7] showed that if f has the asymptotic average shadowing property then it is chain transitive. Fakhari and Ghane [5] proved that if f has the ergodic shadowing property then it is topologically mixing. For that, we study a kind of the shadowing properties which is called limit shadowing property. The limit shadowing property was introduced by Pilyugin *et al*[4]. For a sequence $\{x_i\}_{i \in \mathbb{Z}} \subset X$, the sequence $\{x_i\}_{i \in \mathbb{Z}}$ said to be a *limit pseudo orbit* of f if $d(f(x_i), x_{i+1}) \rightarrow 0$ as $i \rightarrow \pm\infty$. We say that f has the *limit shadowing property* if for a limit pseudo orbit $\{x_i\}_{i \in \mathbb{Z}} \subset X$ there is $y \in X$ such that

$$d(f^i(y), x_i) \rightarrow 0, \text{ as } i \rightarrow \pm\infty.$$

By [15, Example 1.19], the limit shadowing property is not equal to the shadowing property. Carvalho has proved in [2] that if a diffeomorphism belongs to the C^1 -interior of the set of all limit shadowing diffeomorphisms then it is a transitive Anosov diffeomorphism and, in [3], Carvalho and Kwietniak proved that if a diffeomorphism f has the limit shadowing property then it