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PERIODICALLY PERTURBED SUPERLINEAR DUFFING TYPE EQUATIONS WITH SINGULARITIES: SUBHARMONIC SOLUTIONS AND COMPLEX DYNAMICS

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Abstract. We prove the existence of infinitely many subharmonic solutions as well as the presence of a full symbolic dynamics for a class of second-order nonlinear ordinary differential equations with a singularity of repulsive type at the origin and a superlinear growth at infinity. Our result, although dealing with a planar Hamiltonian system, is stable for small perturbations and we can allow the presence of a damping coefficient as well. More general examples of Duffing equations with singularities are discussed, too.

Keywords. Duffing equations, superlinear, singularity, subharmonics, chaotic dynamics.

AMS (MOS) subject classification: 34C25.

1 Introduction

From the time of the pioneering work of Yoshisuke Ueda, who discovered the presence of "chaotic attractors" for the periodically perturbed Duffing equation with a small friction

$$\ddot{x} + c\dot{x} + x^3 = b\cos(t), \tag{1.1}$$

a lot of research has followed, in view of proving the existence of chaotic dynamics for such superlinear and periodically perturbed ODEs. Indeed, in the early sixties, Ueda found chaotic patterns in certain domains of the parameter space (c, b). Famous in this context are the so-called *Japanese Attractor* (for $c = 0.10$ and $b = 12$) and the *Ueda's Chaotic Attractor* (for $c = 0.05$ and $b = 7.5$). Typical features of these equations are the presence of a small friction coefficient along with a nonlinear term with a superlinear growth at infinity (see [1]).

Equation (1.1) is a particular case of a periodically forced superlinear Duffing equation. With this term we mean nonlinear equations of the form

$$\ddot{x} + f(x) = e(t) \tag{1.2}$$

where $f : \mathbb{R} \to \mathbb{R}$ is a locally Lipschitz continuous function satisfying the superlinear growth condition at infinity

$$\lim_{s\to\pm\infty} \frac{f(s)}{s} = +\infty. \tag{1.3}$$