

ON A TWO-DIMENSIONAL SYSTEM OF THIRD-ORDER DIFFERENCE EQUATIONS WITH VARIABLE COEFFICIENTS

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Abstract. We carry out a Lie symmetry analysis of a system of third-order difference equations with variable coefficients. As a result, non-trivial symmetry generators are obtained. Furthermore, we derive exact solutions via the invariants of the group of point transformations.

Keywords. Difference equation, symmetry, reduction, group invariant solutions.

AMS (MOS) subject classification: 39A05, 39A11.

1 Introduction

The Lie group analysis of differential equations is now well-documented. One of its applications is the reduction of order by group theoretic algorithms. Symmetry methods for differential equations have been extended to difference equations [14, 8, 11]. The idea is to solve difference equations by utilising the group of transformations that leave the equation invariant. In [8], the author came up with a systematic algorithm that enables one to find the group of transformations for difference equations. Despite the fact that his method is valid for any given difference equation, he mainly applied it to equations of lower order. This could be attributed to the fact that calculations become highly cumbersome for higher order equations. For solution finding via symmetry analysis, we refer the reader to [9, 5, 6, 12].

Many researchers have studied ordinary difference equations using different approaches [1, 2, 3, 4, 10, 16, 17, 18]. In [18], the authors solved the system of rational difference equations

$$x_{n+1} = \frac{x_{n-1} \pm 1}{y_n x_{n-1}}, \quad y_{n+1} = \frac{y_{n-1} \pm 1}{x_n y_{n-1}}. \quad (1)$$