## AN AGE-INFECTION-STRUCTURED HIV MODEL WITH VERTICAL TRANSMISSION

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**Abstract.** HIV/AIDS is both vertically and horizontally transmitted, and it's progress depends on infection age (duration since infection). According to the characteristics of HIV/AIDS, we study an infection age HIV model with vertical transmission. The population is divided into susceptible and infective, the the susceptible population satisfies ordinary differential equation, and the infective population satisfies partial differential equations for the infection age. We investigate the asymptotic behavior of the steady states. If the basic reproduction  $R_0 < 1$ , the stable infection-free state is globally asymptotically stable. If vertical transmission parameter and horizontal transmission parameters satisfy some conditions, and  $R_0 > 1$ , then the endemic state is asymptotically stable.

**Keywords.** Vertical transmission; infection age; complete orbit; continuous semigroup; asymptotical stability.

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## 1 Introduction

The infection-age-structured HIV model is designed by dividing the population into two compartments containing susceptible individuals and infected individuals. Let S(t) be the number of respectively the susceptible, and  $I(t,\theta)$  be the density of infected population at infection-age  $\theta$  and time t, where infection-age  $\theta$  is the time since the moment of being infected. The compartment of infected population only contains HIV-positive people. The symptomatic individuals (AIDS) have few chances to infect susceptible individuals, and so we assume that they do not transmit HIV to susceptible individuals.

Set

 $\nu$ : birth rate of susceptible individuals, and assuming that the mortality of population is equal to the birth rate of susceptible individuals;  $\nu_1(\theta)$ : extra-mortality due to infection, which depends on infection-age;