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ON THE POLYNOMIAL SOLUTIONS OF GENERALIZED RICCATI EQUATIONS

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Abstract. Consider generalized polynomial Riccati differential equations of the form $a(x)y^m\dot{y} = b_0(x) + b_1(x)y + b_2(x)y^2$ with all the involved functions being polynomials of degree at most d, and with $m \in \{1, 2\}$. We give the polynomial solutions that they can have.

Keywords: Riccati polynomial differential equations, polynomial solutions

AMS(MOS) subject classification:34C05, 34C07, 34C08

1 Introduction and statement of the results

Generalized polynomial Riccati differential equations of the form

$$a(x)y^m\dot{y} = b_0(x) + b_1(x)y + b_2(x)y^2, \tag{1}$$

(here the dot denotes derivative with respect to the independent variable x and $m \geq 0$), appear in all text books of ordinary differential equations as examples of nonlinear equations and in many mathematical and applied problems, see [5, 6, 7] and the references therein.

The main motivation of this paper comes from the works of [1, 2], where the authors present examples of polynomial Riccati differential equations (with $m = 0$ and with degrees at most four of the polynomials a, b, b_1, b_2), having 4 and 5 polynomial solutions. We want to study the maximum number of polynomial solutions that a generalized Riccati differential equation can have. When $m = 0$, $a(x)$ is constant and $b_2 \neq 0$, it was proved in [4] that equation (1) has at most 2 polynomial solutions and that this bound is sharp. If $m = 0$ and $a(x)$ is non-constant with $b_2 \neq 0$ it was proved in [3] that if we denote by $d := \max\{\alpha_i\}$ being α_i the degrees of $a(x), b_0(x), b_1(x), b_2(x)$, respectively, then equation (1) has at most $d + 1$ (resp. 2) polynomial solutions when $d \geq 1$ (resp. $d = 0$) and the bounds are sharp. To the best of our knowledge the question of knowing the number of polynomial solutions of a Riccati