Dynamics of Continuous, Discrete and Impulsive Systems
Series **B:** *Applications & Algorithms* **26 (2019)** 1-21

NECESSARY AND SUFFICIENT CONDITIONS FOR OSCILLATION OF A NEUTRAL DIFFERENTIAL EQUATIONS WITH POSITIVE AND NEGATIVE COEFFICIENTS

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Abstract. In this paper, necessary and sufficient conditions have been obtained for every solutin of

$$[y(t) - py(t-\tau)]' + Q(t)G(y(t-\sigma)) - P(t)H(y(t-\alpha)) = f(t) \qquad (NH)$$

under the condition $0 \leq p(t) < 1$ or $p < 0$ and $p \neq -1$ with the assumption

$$\int_0^\infty f(t)dt < \infty$$

is oscillatory or tends to zero as $t \to \infty$. Moreover, necessary and sufficient conditions have been obtained for every bounded solution of (NH) is oscillatory or tends to zero as $t \to \infty$.

Keywords. Neutral differential equations, nonlinear, oscillations, positive and negative coefficients.

AMS (MOS) subject classification: 34 C 10, 34 C 15, 34 K 11.

1 Introduction

The oscillation theory of delay differential equations has been extensively developed during the past few years. Most of the known results, however, give sufficient conditions for oscillations.

In [1], Nam P. Bhatia has developed some oscillation theorems for differential equations of the type

$$(l(t)x')' + m(t)g(x) = 0,$$

where $l(t), m(t) \in C[0, \infty), l(t) > 0$ and $g(x) \in C(-\infty, +\infty)$. Also in [1], Theorem 1.2 is sort of a comparison theorem which allows to establish Theorem 1.3, which may be considered as a generalization of the many well-known results.