Dynamics of Continuous, Discrete and Impulsive Systems
Series **A:** *Mathematical Analysis* **26 (2019)** 69-88
 http://www.watam.org

CONSTRUCTION OF GENERALIZED PENDULUM EQUATIONS WITH PRESCRIBED MAXIMUM NUMBER OF LIMIT CYCLES OF THE SECOND KIND

Alexander A. Grin[1] and Klaus R. Schneider[2]

[1]Yanka Kupala State University of Grodno
Ozheshko Street 22, 230023 Grodno, Belarus

[2]Weierstrass Institute for Applied Analysis and Stochastics
Mohrenstrasse 39, 10117 Berlin, Germany

Corresponding author Alexander A. Grin email:grin@grsu.by

Abstract. Consider a class of planar autonomous differential systems with cylindric phase space which represent generalized pendulum equations. We describe a method to construct such systems with prescribed maximum number of limit cycles which are not contractible to a point (limit cycles of the second kind). The underlying idea consists in employing Dulac-Cherkas functions. We also show how this approach can be used to control the bifurcation of multiple limit cycles.

Keywords. Generalized pendulum equation, limit cycles of the second kind, limit cycle of multiplicity three, bifurcation behavior, Dulac-Cherkas function.

AMS (MOS) subject classification: 34C05 34C23.

1 Introduction

We consider on a cylinder $\mathcal{Z} := \{(\varphi, y) : \varphi \in [0, 2\pi], y \in \mathbb{R}\}$ the generalized pendulum system

$$\frac{d\varphi}{dt} = y, \quad \frac{dy}{dt} = \sum_{j=0}^{l} h_j(\varphi, \mu) y^j, \quad l \geq 3 \tag{1.1}$$

depending on the real parameter $\mu \in \mathcal{I}$, where $\mathcal{I}$ is some interval. We assume that the functions $h_j : \mathbb{R} \times \mathcal{I} \to \mathbb{R}$, $0 \leq j \leq l$, are continuous, and additionally continuously differentiable and 2π-periodic in the first variable. We denote by f_l the corresponding vector field

$$f_l = \Big(y, \sum_{j=0}^{l} h_j(\varphi, \mu) y^j\Big).$$