*Dynamics of Continuous, Discrete and Impulsive Systems*
*Series* **A:** *Mathematical Analysis* **26 (2019)** 21-52
 http://www.watam.org

# HYPERBOLIC QUASIPERIODIC SOLUTIONS OF U-MONOTONE SYSTEMS ON REIMANNIAN MANIFOLDS

Igor O. Parasyuk[1]

[1]Department of Mechanics and Mathematics,
Taras Shevchenko National University of Kyiv,
64/13, Volodymyrska Street, Kyiv,0 1601, Ukraine
e-mail: pio@univ.kiev.ua

**Abstract.** We consider a time-quasiperiodic Newtonian equation of motion on a Riemannian manifold. The main results concerns the following issues: (a) the existence of a solution taking values in a given bounded domain of configuration space and possessing a bounded derivative; (b) the hyperbolicity of such a solution; (c) the uniqueness and, as a consequence, the quasiperiodicity of such a solution. Our approach exploits ideas of Wazewski topological principle. We use an auxiliary convex function $U$ to introduce the notion of $U$-monotonicity for the system. The $U$-monotonicity property of the system implies the uniqueness and the quasiperiodicity of its bounded solution. The results obtained are applied to study the motion of a charged particle on a unite sphere under the action of time-quasiperiodic electric and magnetic fields.

**Keywords.** Newtonian equation of motion; Quasiperiodic solution; Riemannian manifold; Monotone system; Exponential dichotomy.

**AMS subject classification:Primary: 34C40; 34C27; 37C65; Secondary: 34C12; 34C46; 37C55.**

## 1 Introduction

Let $(\mathcal{M}, \langle\cdot,\cdot\rangle)$ be a smooth complete connected $m$-dimensional Riemannian manifold with metric tensor $\mathfrak{g} = \langle\cdot,\cdot\rangle$, and let $\nabla$ be the Levi – Civita connection with respect to $\mathfrak{g}$. For a given smooth mapping $x(\cdot) : I \mapsto \mathcal{M}$ of an interval $I \subset \mathbb{R}$, denote by $\nabla_{\dot{x}}\dot{x}(t)$ the covariant derivative of tangent vector field $\dot{x}(\cdot) : I \mapsto T\mathcal{M}$ along $x(\cdot)$ at the point $t \in I$. Here $T\mathcal{M} = \bigsqcup_{x\in\mathcal{M}} T_x\mathcal{M}$ stands for the total space of the tangent bundle with natural projection $\pi(\cdot) : T\mathcal{M} \mapsto \mathcal{M}$, and $T_x\mathcal{M} = \pi^{-1}(x)$ denotes the tangent space to $\mathcal{M}$ at $x$.

This paper aims to study a time-quasiperiodic second-order system

$$\nabla_{\dot{x}}\dot{x} = f(t\omega, x), \tag{1}$$