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A NUMERICAL APPROACH TO SOLVE QUADRATIC CALCULUS OF VARIATION PROBLEMS

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Abstract. Several problems in calculus of variations can posses a quadratic integrand or can be approximated by a quadratic function. This paper presents a numerical scheme to solve the quadratic calculus of variations problem. The Legendre wavelets operational matrix of derivative is used to reduce this problem to a static quadratic programming, which allows us to deal with large-scale problems. Illustrative numerical tests are included to demonstrate the efficiency and applicability of the technique. Numerical comparison is also given with others method from the state-of-the-art. The formulation presented is simple and can be naturally extended to nonlinear calculus of variations problems.

Keywords. Calculus of variations, Legendre wavelets, Quadratic programming, Operational matrix of derivative, Active-set method.

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1 Introduction

Let $\mathcal{L}$ be a real valued function defined on $[0,1] \times C^1_{([0,1],R^n)} \times C^0_{([0,1],R^n)}$. The calculus of variations problem is the problem of finding the extremum of the functional

$$J = \int_0^1 \mathcal{L}\left(t, x\left(t\right), \dot{x}\left(t\right)\right) dt \tag{1}$$

with some boundary conditions

$$(x\left(0\right) = A_1 \text{ or } \quad \dot{x}\left(0\right) = B_1) \quad \text{ and } \quad (x\left(1\right) = A_2 \text{ or } \quad \dot{x}\left(1\right) = B_2), \tag{2}$$

In this paper we are interested by the numerical solution of this problem, when the integrand function $\mathcal{L}$ is quadratic, i.e;

$$\mathcal{L}\left(t, x\left(t\right), \dot{x}\left(t\right)\right) = v^{\top}\left(t\right) H v\left(t\right), \tag{3}$$