Dynamics of Continuous, Discrete and Impulsive Systems
*Series **B:** Applications & Algorithms* **25 (2018)** 357-367

A BLOW-UP RESULT IN A CAUCHY VISCOELASTIC PROBLEM WITH A DELAYED STRONG DAMPING

Mohammad Kafini[1], Muhammad I. Mustafa[2]

[1]Department of Mathematics and Statistics
KFUPM, Dhahran 31261, Saudi Arabia
E-mail: mkafini@kfupm.edu.sa

[2]Department of Mathematics, University of Sharjah, Sharjah, UAE
E-mail: mmustafa@sharjah.ac.ae

Abstract. In this paper we consider a Cauchy problem for a nonlinear viscoelastic equation with delay in the strong damping. Under suitable conditions on the initial data and the relaxation function, we prove a finite-time blow-up result.
Keywords: Blow up, Cauchy problem, delay time, relaxation function, viscoelastic.
AMS Classification: 35B05, 35L05, 35L15, 35L70

1 Introduction

In this work, we are concerned with the following delayed Cauchy problem

$$\begin{cases} u_{tt} - \Delta u + \int_0^t g(t-s)\Delta u(x,s)ds + \mu_1 \Delta u_t(x,t) + \mu_2 \Delta u_t(x,t-\tau) \\ = |u|^{p-1} u, \quad x \in \mathbb{R}^n \quad , t > 0 \\ u_t(x,t-\tau) = f_0(x,t-\tau), \text{ in } (0,\tau), \\ u(x,0) = u_0(x), \quad u_t(x,0) = u_1(x), \quad x \in \mathbb{R}^n, \end{cases} \tag{1}$$

where $p > 1$, μ_1 is a positive constants, μ_2 is a real number, and $\tau > 0$ represents the time delay. The function g is the relaxation function subjected to conditions to be specified and u_0, u_1, f_0 are the initial data to be specified later. The space of the problem here is the whole space of $\mathbb{R}^n$.

Viscoelastic wave problems in bounded domains were considered by many authors. Messaoudi in [1] considered the following initial-boundary value problem

$$\begin{cases} u_{tt} - \Delta u + \int_0^t g(t-\tau)\Delta u(\tau)d\tau + u_t|u_t|^{m-2} = u|u|^{p-2}, \quad \Omega \times (0,\infty) \\ u(x,t) = 0, \quad x \in \partial\Omega, \ t \geq 0 \\ u(x,0) = u_0(x), \quad u_t(x,0) = u_1(x), \quad x \in \Omega, \end{cases} \tag{2}$$

where Ω is a bounded domain of $\mathbb{R}^n$ $(n \geq 1)$ with a smooth boundary $\partial\Omega$, $p > 2$, $m \geq 1$, and $g : \mathbb{R}^+ \longrightarrow \mathbb{R}^+$ is a positive nonincreasing function. He showed, under suitable conditions on g, that solutions with initial negative energy blow up in finite time if $p > m$ and continue to exist if $m \geq p$. This